AN EXPLORATORY APPROACH TO THE FORMULATION OF INPUT-OUTPUT MODELS AND THE APPLICATION OF INPUT-OUTPUT PROCEDURES IN EDUCATIONAL COST MODELLING

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THESIS

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ABSTRACT

A general cost model for an educational institution is formulated. This model is developed by applying classical Leontieff input-output procedures in a situation where multiple outputs and resource inputs exist. The expansion of the classical Leontieff model to the tableau format of the "Electric" Five Year Defense Plan is presented. The general assumptions relating to input-output models are presented and analyzed in the general educational setting. To provide an example of possible uses of the model, it is applied to the Naval Postgraduate School and limited empirical results are presented. Shortcomings revealed during this first empirical use of the model are discussed.

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I. INTRODUCTION

Recent increased use of input-output models as predictive cost models has resulted in a growing interest about the interworkings of these type models. This thesis represents the author's research in support of the current efforts of several professors at the Naval Postgraduate School relating to conceptual studies being conducted by the Department of the Navy and the Naval Postgraduate School. The two study areas most directly related to this effort are the Navy's (OP-96) construction of an input-output cost model for the Navy and Marine Corps and the Postgraduate School's requirement for a better tool to aid the School's managers in the prediction of the resources required for the operation of the School. Thus it is of value to develop an input-output cost model of such a size and complexity that it is possible to apply the model to an organization and analyze the results. Proceeding thus relates directly to the Navy's studies and may in fact result in procedures that can be adapted for use at the Postgraduate School. The actual model construction should lead to a better understanding, and thus increase knowledge, relative to the development of model parameters, means of aggregation of variables, interpretation of variables, and other such areas.

It is envisioned presently that the total research effort will be separated into two phases. Phase I will be

directed at the general development of procedures for the use of input-output techniques in cost modelling and to attempt a first empirical application of these procedures to a specific organization. The application of the input-output technique should lead to the development of and possible interpretation of a specific set of variables, definition of measures of output and input, and the gathering and use of actual data in a first attempt at empirical application. The final stages of phase I will be devoted to using the forecasting errors, experience developed in constructing the model, and any subsequent developments which occur outside the Naval Postgraduate School to develop appropriate additional research strategies for refinement of the procedures prior to moving to phase II. This thesis specifically relates to phase I of the total research effort.

To support the research strategy outlined above, it was decided to develop a general input-output cost model for an educational institution and then to attempt to apply the procedures specifically to the Naval Postgraduate School. This approach was chosen since, as already mentioned, the Postgraduate School was interested in developing a better means of forecasting resource requirements, the School was of such a size that it was believed that an input-output model (for the Postgraduate School) would not be too large, and data for past resource usage was available.

It is intended that phase II will be concerned with looking at different operational time periods (the initial period was the fiscal year) for the model, specifically a quarter (or twelve week) time period which is the time frame in which the Postgraduate School actually operates; the effect of applying different rules in the aggregation of variables; and the feasibility of using the model to assist the Postgraduate School with its segmentation problem. The segmentation problem is the grouping of students into sections or classes for instructional purposes. The only portion of this thesis relating to phase II directly is the final chapter.

Since the primary objective of this thesis results in the construction of an input-output model that will be for an educational institution it seems appropriate to provide a brief discussion relating to the economics of education. In general, study within the area of educational economics has been directed in two primary areas. The first area has been concerned principally with the analysis of the economic value of education. Efforts in this field have related to determination of the contribution of education to economic growth through investigation of the impact of schooling on labor productivity, occupational mobility, and distribution of income. The second area of interest has been concerned with the analysis and description of the economic aspects of educational systems. This latter area has required the study of the internal efficiency of

the school and the relations between the costs of education and the financing of these costs.[1]

The educational cost model developed in this thesis in the context of the educational economics just discussed is related to the second area of interest. It is, however, often necessary to consider the economic value of education as the first step in the economic analysis of an educational system. This sequence is essential as it is an output value that is measured ultimately in the application of input-output methods. In many cases, as discussed in the following chapter, the output measure is in fact a "value-added" or "net-benefit" measure that occurs in conjunction with the educational process. This analytic description of the school will allow school administrators and academic planning councils to analyze the cost and resource implications of alternative levels and mixes of faculty, students, facilities, and other resources available to the school.

The next chapter, Chapter II, is a discussion of various approaches to the general modelling of educational institutions. It is included to provide information on current and past research and to discuss some of the advantages and disadvantages of various approaches. This chapter provides survey information for users relating to other types of educational models. Chapter III contains a discussion of the applicable mechanics of input-output

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[1] Blaug, M., editor, Economics of Education 1, p. 8, Penguin Books, 1968.

analysis and presents the development of the general cost model. This chapter, Chapter III, further contains a discussion of the assumptions relating to the model, either in the general case or in a specific application such as to the Naval Postgraduate School, and a discussion of one means of validation. Chapter IV contains the specific application of the procedures to the Naval Postgraduate School. The final chapter contains a summary of this author's research effort and some suggested research areas for future work. Thus the final chapter primarily relates to areas to be considered in phase II of the total research effort. It should be noted, however, that some of the items discussed in this final chapter are relevant to refinements that must be accomplished prior to passing into phase II.

II. EDUCATIONAL MODELS

The degree of complexity of educational models varies from simple aggregate average cost models to complete simulations of the educational institution. In the simplest case, an average cost per student per year, quarter, credit hour, or such is computed. These averages are based on aggregating the entire faculty of an academic department and all students taught by that same faculty into two categories, faculty and students. This aggregation means the loss of information such as explicit identification of students by degree levels such as B.S., B.A., or M.S. Also lost is delineation of specific faculty departmental instruction to particular student degree areas such as the number of classroom instruction hours provided by a mathematics department to mechanical engineering students, sociology students, or management students. In computing the average cost per student in this manner the overhead costs associated with the educational institution are generally computed on a pro-rata share. These costs relate to such things as janitorial services, repair and maintenance of facilities, library and computer center operations, or the provision of medical and dental services. The shortcoming of this type of approach results from a failure to delineate explicitly the many interrelations that exist among the various academic departments, the students, and the educational support activities such as

libraries, computer facilities, and other similar type activities. In this context the average cost per student fails to provide the school administration with a tool to aid in planning or decision making. The only value of such average cost figures would therefore seem to be as a device to indicate changes in total student educational costs on a year to year basis. Any in-depth analysis of where or why major changes have occurred still requires a detailed, and hence non-aggregated, investigation of the components which comprise the total average cost.

A second type of educational model is designed to measure the "value-added" or "net-benefit" between the input and output of the system. One approach to analyzing educational models of this type is to consider a school to be structured into demand elements and service units which are linked together through a set of rules and operating procedures which describe the transformation of an input into an output. The demand element can most easily be identified by considering the function or purpose of the educational institution. One goals structure, or objective, for a school discussed by Hamelman and Mazze consists of three broad categories for the classification of output of a school. In this classification scheme the school's outputs are considered to be classroom instruction, research by the academic departments, and community service activities.[2] This structure

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[2]Hamelman, P. W., and Mazze, E. M., A Systems View of a University: The Missions Concept of Planning-Programming-Budgeting, paper presented at Eastern Academy of Management Meeting, University of Massachusetts, April 1970.

is used in this thesis. Specific examples of output measures for this goals structure might include the number of students at various degree levels (B.S., M.S., Ph.D.) or within academic discipline areas (Math, Engineering, Physics); the number of published articles, honorary awards, or possibly the amount of reference made to a faculty member in the literature; and services relating to medical, dental, or counseling problems. The service units can be classified primarily as the resources available to the school less the students. Thus the faculty, classrooms, laboratories, grants, and other such resources comprise the service population. It is possible for a student to be an element of this population as well as the demand population. Some simple examples of this are interns, student nurses, or teaching assistants. Rules and operating procedures serve two distinct purposes in the present context. First, they provide the guidelines and directions for the allocation and assignment of the service units to the demand population. An example of this function is the specification and sequencing of courses required for the award of different degrees. Second, the specification of rules and procedures is necessary for measuring and evaluating system performance. The requirement to pass successfully a comprehensive examination prior to the award of a degree might be an example of a means of evaluating the educational institution's performance. Educational planning models which employ this concept (value-added) can be classified generally into one of four basic functional areas.

Enrollment population models include information relating to the number and types of students. This information may be as general as the number of students at the B.S., M.S., or Ph.D. level or as specific as the number of students at the M.S. level in mathematics specializing in boolean algebra and numbers of students in other academic areas with the same amount of specificity. Also included in this area is information relating to the dynamics of student-flow through the educational institution. Mandatory subjects and acceptable elective sequences required for the award of a specific degree represent an example of this type of flow-dynamics.

Resource requirements models are used to forecast the school's current and projected requirements for instructors, classrooms, laboratories, funds, and the like. The determination of the appropriate level is based upon current and projected student input and any other changes in the school's goals structure such as increased research effort or expanded services to society.

Allocation and scheduling of resources models are used to accomplish the linking of the demand elements and service units. One side of this process contains all the students and their respective course requirements while the other side includes the classrooms, laboratories, and instructors available to the school to accomplish its mission. Individual preferences of students as related to sequencing and class times may be allowed to a certain extent. In the same

manner the individual preferences of faculty members may be expressed, and school policies relating to class sizes, classroom and laboratory utilization, and other similar administrative matters may be inserted into the process. The eventual output of this process is a schedule for the students and faculty.

The final functional area for an educational planning model in the present context is related to the evaluation of the system performance. These models are used for review of specific educational programs, strategies, or system configuration. They are in a sense "yardsticks" in that they serve to compare an actual accomplishment or output with that which was projected as being desired. In other words they represent a comparison between a realization and a forecast. These models also include related cost-benefit and cost-effectiveness measures in the evaluation scheme.

The analysis of educational planning models in the manner just described is being investigated by the Western Higher Education Institutions and Agencies. The basic design of their model includes submodules for student instruction, research, and external service.[3] Each of these submodules uses faculty, physical facilities, supplies and equipment, research, external service activities, library, and other academic services as system inputs. The difference between

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[3]A submodule is a grouping of those activities of the institution that comprise one element of the goals structure for the school. Each submodule is a complete entity and hence a major component of the total model.

the value of the input and the value of the output is considered to be the "value-added" or "net-benefit" to the student, knowledge, or society.[4] The strength of a system of this nature is that it provides a complete model or description of the interrelations of the educational institution when all submodules are linked together through an executive or control system. The complex nature of the functions that must be accomplished in describing student enrollment, determining resource requirements, allocating and scheduling resources, and evaluating system performance makes it extremely unlikely that this type of model can ever be implemented without the aid of a computer.[5]

A similar, though more extensive model, is being developed by the University of Texas with their "Project Generalized University Model."[6] It is anticipated that

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[4]Western Interstate Commission for Higher Education draft copy, Subject: A Proposal to Design and Implement a Management Information System with Common Data Elements for Western Higher Education Institutions and Agencies, 1 February 1968.

[5]An excellent example of the magnitude of this problem, to include discussion of the computer programming effort, has been prepared by the Virginia Polytechnic Institute. The reader is referred to -

Lee, Sang M., and Clayton, Edward R., A Goal Programming Model for Academic Planning, Working Paper Number 27 of the Department of Business Administration Virginia Polytechnic Institute and State University prepared for presentation at the Eleventh American Meeting of the Institute of Management Sciences, Los Angeles, California, October 1970.

[6]University of Texas Graduate School of Business final report, Project Generalized University Model Phase III, by T. W. Ruefli and others, November 1969.

this study will lead to a simulation of the entire university. It is currently planned that the simulation will consist of flow models to represent passage of items through an educational process, tactical models to deal with operational decisions, and planning models which will be primarily long-range but will be capable of handling problems that may develop on a day to day basis. The flow models will represent student, faculty, and facilities. The tactical models will accomplish such actions as assignment of faculty and students to courses, facility assignment, administrative functions relating to admission, continuance and probation, operational budget actions, and computer scheduling. Finally the planning models will relate to financial affairs, facility usage, design of new university facilities, expansion of existing facilities, curriculums, research, personnel actions, land-use, organizational structure, and portfolio management. The flow models, though primarily concerned with the present situation, can be designed to project into the future. Tactical models will consider the ensuing time period, and planning models will be future-oriented with a limited capability to deal with problems that arise on a daily basis.[7] The complexity and detail of this system equals or exceeds that of the first system discussed in every aspect. Current plans call for the system to be completely computerized. The

[7] Ibid.

magnitude of the effort expended in the development of this system has been significant.[8] It is doubtful that many institutions can afford to expend the time, money, and talent to develop such a detailed and complete model of their activities. The final product, as in the previous case, will be a total system model of the university and its various activities.

Application of input-output procedures provides a possible alternative approach to the modelling of an educational institution. These procedures allow measurement of outputs in terms of classroom instruction, research, and public service and provide explicit delineation of the interactive relations that exist in the school.[9] In contrast to the two systems just discussed it is not necessary that this approach be computerized although it may be. Extensive research on the use of input-output procedures for cost modelling has been conducted within the

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[8] The labors expended by the University of Texas have included the efforts (part time in some cases) of one dean, two full professors, one project director (an assistant professor), nine research associates, and one administrative assistant. Ibid.

[9] The three output areas listed here are those previously mentioned in conjunction with a school's goals structure. These are not the only outputs that may be used but are considered to be representative of most output measures. The interactive relations of a school refer to the associations among the various academic departments, support elements such as libraries, and outputs.

Department of Defense. These efforts have included the actual construction of input-output models for some Department of Defense activities as early as 1969.[10] Since this time the Army has constructed an input-output model which was used in conjunction with the February 1971 tentative fiscal guidance and for construction of computer tapes used in conjunction with program objective memoranda exercises. Also, the Center for Naval Analysis is presently building an input-output model for the Navy (to include the Marine Corps). This model will be used by the Navy in future planning exercises. Information relating to the status of input-output usage within the Army and the Navy has been obtained through communications between the author and Army officials.

The adaptation of the input-output procedures developed within the Department of Defense to the formulation of a cost model for an educational institution requires, for the most part, only minor changes to the existing standard input-output framework. These procedures, input-output, do not provide the extensive total system model being investigated by the Western Higher Education Institutions and Agencies or the University of Texas. They do, however, provide a means for developing possibly better cost forecasts by those school administrations

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[10] Proceedings of the Fourth Annual Department of Defense Cost Research Symposium, The Marine Corps Cost Model, by J. H. Augusta and R. A. Jenner, p. C-72, 17-18 March 1969.

that are not able to devote the resources to the development of a total system. In this context the input-output model is considered to be a possible replacement for the costing procedures presently used by an educational institution. The mechanics of application of input-output procedures and their use in construction of a cost model for the Naval Postgraduate School will be the subject of the remainder of this thesis.

III. THE GENERAL MODEL AND RELATED INPUT-OUTPUT MATHEMATICS

Cost models may be categorized broadly as being of a descriptive or prescriptive nature based upon the purpose or goal of the model, the structure of the model, the applied use of the model, and other related items. Within this general framework it is useful to classify these models in terms of their basic use. One such classification scheme or taxonomy has been proposed by C. R. Jones, and will be used in the following discussion.[11] This taxonomy identifies a cost model as being a model of pure logical consistency, pure explanation, pure prediction, or causal-explanation prediction. A model of pure logical consistency only deals with the model's logical consistency. It addresses the question of whether or not certain "things" logically lead to certain other "things." There are no empirical tests required to check this type of model. A model of pure explanation of existing observations portrays an existing set of observations. The average cost per student model discussed in the early part of Chapter II is representative of this type of model. These models are often of a statistical nature and hence empirical verification procedures do exist. The pure prediction model generates forecasts or predictions for a

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[11] Jones, C. R., A Taxonomy for Naval Force Level and Structure Models, working papers Naval Postgraduate School, Department of Operations Research and Administrative Sciences, Monterey, California, April 1971.

subsequent time period. Empirical verification of this class of models may be accomplished by comparison of forecast and realized observations. The educational institution cost model presented in this thesis is a member of this class of models. Specific reference to verification of the school model will be presented later in this Chapter. The final class of models, causal-explanation prediction, represents the most sophisticated type of model within this taxonomy. This model is again a predictive model. However, in this case, prediction is based on an understanding of the existing process in terms of causality.[12] It is hoped that input-output models of the type proposed in this thesis will pass from the pure prediction class to the causal-explanation prediction class when scientific knowledge progresses to the stage that it is possible to physically describe the process being analyzed. This author believes it is presently questionable if any existing cost model can be considered to be a member of this class. This brief discussion of classification schemes has been included to present a general framework that relates to the basic uses of cost models. The remainder of this Chapter will relate to the use of a modified Leontieff input-output model as a pure predictive cost model for an educational institution.

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[12]As used here causality is taken to mean that it is possible to relate an outcome to the interaction of one or more variables. A much more extensive discussion is contained on pages 18 and 19 of the paper referred to in footnote 11.

The classical Leontieff input-output model consists of three basic elements. A square matrix is used to represent the industrial classification of an economy. Each row of this matrix contains the amount of a particular industry's final product that is used as input for producing the final product of the industry in question and all other industries in the economy. A single row vector is used to denote the amount of a single primary factor (input), such as labor required by each industry in the economy. Finally, a column vector is used to provide the amount of exogenous demand that exists for each final product.[13] As a simplified example, consider a two-industry economy in which industry one produces electricity and industry two produces steel. If x_{ij} denotes the quantity of the i^{th} industry's final product required by the j^{th} industry, then x_{11} represents the amount of electricity required in the production of electricity and x_{12} represents the amount of electricity used in the production of steel. In a physical sense x_{11} represents the electricity used to operate the lights and other electrical devices in the dams, steelmills, and other related activities of the two industries. In a similar manner, x_{21} and x_{22} represent the amount of steel used in construction and related activities in the electricity and steel industries respectively. Together x_{11}, x_{12}, x_{21}, and x_{22}

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[13] Dorfman, Robert, Samuelson, P. A., and Solow, R. M., Linear Programming and Economic Analysis, p. 2?4-208, McGraw-Hill, 1958.

form the square matrix described above. Since this example is a two-industry economy the amount of labor required can be represented by x_{01} and x_{02} and the exogenous demand for final products by d_1 and d_2. Figure 1 depicts the general model for an n-industry economy.

The cost model used for the educational institution is an expansion of the classical model just discussed and portrayed in Figure 1. In the model for the school the single vector of exogenous demand is expanded to include a vector for each output of the school. For the goals structure presented in Chapter II this means there will be a series of vectors for the outputs of student instruction, research, and contributions to society. Also, the single primary factor (input) of labor is replaced by budgetary information relating to manpower and expenditures. This model (of the school) is composed of four sub-elements and is depicted as shown in Figure 2. Section I of the Figure is the traditional square matrix depicting the interrelations of the school. It lists, for example, the amount of support provided by the library or computer center to the academic departments, administrative units, and all other elements that make up the school. This support may be measured in terms of designated work units such as number of issues or number of jobs processed or it may be measured in terms of the amount of time devoted to accomplishing work tasks for a particular element. All elements of the school required for the production of output are portrayed as a row and a column in this section of the model. Section II of the

	Industry 1	Industry 2		Industry j		Industry N	Final Consumption	Total Output
Industry 1	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1N}	d_1	x_1
Industry 2	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2N}		
	$\vdots$			$\vdots$		$\vdots$		
Industry i	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{iN}	d_i	x_i
	$\vdots$			$\vdots$		$\vdots$		
Industry N	x_{N1}	x_{N2}	$\cdots$	x_{Nj}	$\cdots$	x_{NN}	d_N	x_N
Labor	x_{01}	x_{02}	$\cdots$	x_{0j}	$\cdots$	x_{0N}		

Figure 1. The Classical Leontieff Model.

SECTION I Input-Output Relations	SECTION II Vectors of Outputs
SECTION III Expenditures and Personnel Assets Devoted to School (Internal)	SECTION IV Expenditures and Personnel Assets Devoted to Each Output

Figure 2. Schematic of the School Model.

Figure contains a series of column vectors, one for each output of the school. Each of these vectors contains information relating to the amount of input required in producing a given level of output. This portion of the model thus depicts the exogenous demands for the open model as mentioned above. Section III of the Figure represents the expansion of Leontieff's primary factor of input. An entry in this section corresponds to the expenditure of a certain amount of funds or allocation of personnel spaces for the applicable school element listed in the columns of Section I of the model. Section IV of the Figure contains the same budgetary-type information for expenditures and manpower allocation related to the outputs of the production process. Consideration of the type of information contained in Sections III and IV of the model reveals that these Sections represent, respectively, the indirect and direct costs of operating the school. Those familiar with the current construction of force-cost models within the Department of Defense will recognize immediately that this tableau (Figure 2) is of the form of the so-called "Electric" Five-Year Defense Plan.[14] The manner in which a school is cast into this specific tabular form will be postponed until the assumptions related to the model and the mathematics applicable to the input-output model are presented. This approach

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[14]The term, tableau, will be used in the remainder of this thesis to denote an input-output model of the configuration shown in Figure 2.

is taken since the constraints imposed as a result of the assumptions relating to the model and the applicable mathematics directly affect the school's tabular formulation.

It is important to remember that the model depicted in Figure 2 has simply replaced the single output and single primary factor of input of Figure 1 with a vector in each case. Hence, all the traditional assumptions relating to Leontieff input-output models remain applicable in the modified configuration of Figure 2. These assumptions relate to output measures, returns to scale, and the transformation process of inputs into outputs. Each element in Section I of Figure 2 is assumed to have a single output measure. Examples of appropriate measures for a school might include the amount of computer time furnished by a computer activity to other school elements, the amount of classroom or laboratory contact hours expended by an academic department, or the number of issues and external transactions completed by a library. With reference to returns to scale, it is assumed that these are constant. This means that if all inputs are doubled, tripled, or increased k-fold, then all outputs are increased by the same respective amounts. Finally, the transformation of inputs into outputs is assumed to be accomplished in a linear fashion and, even more importantly, it is assumed that the input-output coefficients are constant. These assumptions will be discussed in greater detail following the explanation of the mathematics of the input-output model.

Figure 3 portrays the general tableau for a school which has n-inputs, m-outputs, and ℓ-categories of expenditure and manpower assets of interest. The tabular entries are defined in Table I. To introduce the method of working with an input-output model the solution to the two-industry economy problem previously introduced will now be presented. For this example the solution procedure deals with the problem of determining the appropriate level of output for each of the two industries to operate at so that production just satisfies the total demand for a final product. In other words, equilibrium exists in this two-industry economy when the total outputs of industry one and two are in balance in the sense that just enough of each is produced to satisfy both the final demand and the input requirements for each product. Again, in the general case, the two-industry economy becomes an n-industry economy and the rest of the above discussion remains the same. Proceeding with the solution to the two-industry problem let x_1 and x_2 represent the total output requirement of industry one and two respectively. These quantities are row sums in terms of Figure 1. From the discussion of the two-industry economy and the assumptions relating to the input-output model it follows that for each unit of the j^{th} commodity produced a fixed input of the i^{th} commodity is required. In other words, a unit output respectively of electricity or steel requires a fixed input of electricity and steel. These fixed requirements are the constant input-output coefficients referred to in the final assumption

Table I. Definition of Tabular Entries for Figure 3

$\overline{SX}_{ij}$	level of input from the i^{th} activity to the j^{th} activity.
M_i	total intermediate requirement for the i^{th} activity's input. This is consumed within the school.
$\overline{GL}_{ik}$	level of input from the i^{th} activity to the k^{th} output.
N_i	total final requirement for the i^{th} activity's input. This is consumed by an output activity.
X_i	total requirement for the i^{th} activity's input. This is the sum of M_i and N_i, the intermediate and final requirements.
$\overline{BX}_{tj}$	level of resource t required to produce the j^{th} output.
O_t	total intermediate requirement for the t^{th} resource.
$\overline{DL}_{tk}$	level of resource t required to produce the k^{th} output.
P_t	total final requirement for the t^{th} resource.
E_t	total requirement for the t^{th} resource. This is the sum of O_t and P_t, the intermediate and final requirement for the t^{th} resource.
L_k	level of physical output for the k^{th} output activity.

$$
\begin{array}{lcl|lcl|l}
x_1 \quad x_2 \quad \cdots \quad x_n & & & L_1 \quad L_2 \quad \cdots \quad L_m & & & \text{TOTAL} \\
\hline
\overline{SX}_{11}+\overline{SX}_{12}+ \ \cdots \ +\overline{SX}_{1k}+ \ \cdots +\overline{SX}_{1n} & = & M_1 & \overline{GL}_{11}+\overline{GL}_{12}+ \cdots\cdots +\overline{GL}_{1m} & = & N_1 & X_1 = M_1 + N_1 \\
\overline{SX}_{21}+\overline{SX}_{22}+ \ \cdots & & & \overline{GL}_{21}+ & & & \\
\vdots \qquad \overline{SX}_{ij} & & & \vdots \qquad \overline{GL}_{ik} & & & X_i = M_i + N_i \\
\overline{SX}_{n1}+\overline{SX}_{n2}+ \cdots\cdots +\overline{SX}_{nn} & = & M_n & \overline{GL}_{n1}+ \cdots\cdots +\overline{GL}_{nm} & = & N_n & X_n = M_n + N_n \\
\hline
\overline{BX}_{11}+\overline{BX}_{12}+ \cdots\cdots +\overline{BX}_{1n} & = & O_1 & \overline{DL}_{11}+\overline{DL}_{12}+ \cdots\cdots +\overline{DL}_{1m} & = & P_1 & E_1 = O_1 + P_1 \\
\vdots \qquad \overline{BX}_{tj} & & & \vdots \qquad \overline{DL}_{tk} & & & \\
\overline{BX}_{\ell 1}+\overline{BX}_{\ell 2}+ \cdots\cdots +\overline{BX}_{\ell n} & = & O_\ell & \overline{DL}_{\ell 1}+ \cdots\cdots +DL_{\ell m} & = & P_\ell & E_\ell = O_\ell + P_\ell
\end{array}
$$

Figure 3. The General School Tableau.

relating to the model. These coefficients for this problem are $a_{11} = x_{11}/x_1$, $a_{12} = x_{12}/x_2$, $a_{21} = x_{21}/x_1$, and $a_{22} = x_{22}/x_2$. In general these coefficients are defined by $a_{ij} = x_{ij}/x_j$. The total output requirements, x_1 and x_2, may now be expressed as:

$$x_1 = a_{11}x_1 + a_{12}x_2 + d_1$$

$$x_2 = a_{21}x_1 + a_{22}x_2 + d_2.$$

Proceeding to solve these two equations yields:

$$d_1 = (1-a_{11})x_1 - a_{12}x_2$$

$$d_2 = -a_{21}x_1 + (1-a_{22})x_2.$$

These expressions can be more clearly expressed in matrix notation as follows:

$$\begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}.$$

This last expression can be more compactly written as

$$(I-A)\overline{x} = \overline{d}.$$ [15]

Direct solution of this last result for $\overline{x}$ completes the problem,

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[15] This type notation will be used throughout the remainder of this thesis. A will denote the matrix $[a_{ij}]$, $\overline{x}$ the vector of outputs, $\overline{d}$ the vector of exogenous demands, and I the identity matrix,

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \\ 0 & 0 & 0 & & & & \end{bmatrix}$$

and it follows that $\bar{x} = (I-A)^{-1}\bar{d}$.[16] It can be seen that the appropriate level of output for each of the two industries is uniquely determined for a given exogenous demand.

The solution of the mathematical problem portrayed in Figure 3 is the determination of a budget forecast, an E vector as defined in Table I, for a given output level and mix. The solution to this problem is basically the same as the solution to the classical two-industry economy Leontieff problem just discussed. The only changes occur as a result of having a vector of outputs that determines the exogenous demand and a two-component primary input which specifies dollar expenditures in addition to personnel or labor inputs. The first step of the solution is the determination of the total requirement for the i^{th} activity's input. This is determined by using historical budgetary data for a known output level and mix. Once this has been determined the input requirements for any other output level and mix may be determined. The second step of the solution simply involves the use of the now

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[16] The question of the existence of a matrix inverse is always of concern. It has been shown that when $a_{ij} \geq 0$ and $\sum_{i=1}^{n} a_{ij} < 1$ $(i,j = 1,2,\ldots,n)$ then the inverse of the matrix $[I-a_{ij}]$ will exist. The first condition will always be satisfied as the physical process considered in this thesis requires that the a_{ij} be non-negative for all i and j. The second condition may or may not be satisfied for any particular institution, and can be verified only after the school has been cast in the format of this model. See

R. M. Solow, "On the Structure of Linear Models," Econometrica, v. 20, p. 29-46, January 1952.

known input requirements in determining the total intermediate and final resource requirements. From Figure 3 and Table I it may be seen that $X_i = M_i + N_i$ or,

$$X_i = \sum_{j=1}^{n} \overline{SX}_{ij} + \sum_{k=1}^{m} \overline{GL}_{ik} \qquad (\text{for } i = 1,2,\ldots,n).$$

Since

$$\sum_{j=1}^{n} \overline{SX}_{ij} = \sum_{j=1}^{n} S_{ij}\ X_j \qquad (i = 1,2,\ldots,n)$$

and

$$\sum_{k=1}^{m} \overline{GL}_{ik} = \sum_{k=1}^{m} G_{ik}\ L_k \qquad (i = 1,2,\ldots,n)$$

it follows that,

$$X_i = \sum_{j=1}^{n} S_{ij}\ X_j + \sum_{k=1}^{m} G_{ik}\ L_k \quad (\text{for } i = 1,2,\ldots,n). \quad (1)$$

This last expression can be portrayed more descriptively as,

$$\underset{n\times 1}{\begin{bmatrix} X_i \end{bmatrix}} = \underset{n\times n}{\begin{bmatrix} S_{ij} \end{bmatrix}} \underset{n\times 1}{\begin{bmatrix} X_j \end{bmatrix}} + \underset{n\times m}{\begin{bmatrix} G_{ik} \end{bmatrix}} \underset{m\times 1}{\begin{bmatrix} L_k \end{bmatrix}} . \qquad (2)$$

Now, by denoting the identity matrix $[I]_{n\times n}$, the solution of expression (2) for X_i proceeds as follows.

$$\begin{bmatrix} I \end{bmatrix}_{n\times n} \begin{bmatrix} X_i \end{bmatrix}_{n\times 1} - \begin{bmatrix} S_{ij} \end{bmatrix}_{n\times n} \begin{bmatrix} X_i \end{bmatrix}_{n\times 1} = \begin{bmatrix} G_{ik} \end{bmatrix}_{n\times m} \begin{bmatrix} L_k \end{bmatrix}_{m\times 1}^{17}$$

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[17]This expression may be written in this manner since x_j is used to denote the transpose of x_i

$$\left\{ \begin{bmatrix} I \end{bmatrix}_{nxn} - \begin{bmatrix} S_{ij} \end{bmatrix}_{nxn} \right\} \begin{bmatrix} X_i \end{bmatrix}_{nx1} = \begin{bmatrix} G_{ik} \end{bmatrix}_{nxm} \begin{bmatrix} L_k \end{bmatrix}_{mx1}$$

$$\begin{bmatrix} X_i \end{bmatrix}_{nx1} = \begin{bmatrix} I - S_{ij} \end{bmatrix}^{-1}_{nxn} \begin{bmatrix} G_{ik} \end{bmatrix}_{nxm} \begin{bmatrix} L_k \end{bmatrix}_{mx1} \tag{3}$$

This completes the first step of the solution as Equation (3) provides the expression for solving for the activity input requirements for a desired output level and mix. Now returning once again to Figure 3 and this time looking at the lower portion of the model it is evident that $E_t = O_t + P_t$. This is more clearly expressed as,

$$E_t = \sum_{j=1}^{n} \overline{BX}_{tj} + \sum_{k=1}^{m} \overline{DL}_{tk} \qquad (\text{for } t = 1,2,\ldots,\ell).$$

Again, as before,

$$\sum_{j=1}^{n} \overline{BX}_{tj} = \sum_{j=1}^{n} B_{tj} X_j \qquad (\text{for } t = 1,2,\ldots,\ell)$$

and

$$\sum_{k=1}^{m} \overline{DL}_{tk} = \sum_{k=1}^{m} D_{tk} L_k \qquad (\text{for } t = 1,2,\ldots,\ell).$$

Substitution of this identity relation yields,

$$E_t = \sum_{j=1}^{n} B_{tj} X_j + \sum_{k=1}^{m} D_{tk} L_k \quad (\text{for } t = 1,2,\ldots,\ell). \tag{4}$$

Once again expressing this result as in the development of (2) it follows that,

$$\begin{bmatrix} E_t \end{bmatrix}_{\ell x1} = \begin{bmatrix} B_{tj} \end{bmatrix}_{\ell xn} \begin{bmatrix} X_i \end{bmatrix}_{nx1} + \begin{bmatrix} D_{tk} \end{bmatrix}_{\ell xm} \begin{bmatrix} L_k \end{bmatrix}_{mx1} \qquad (5)$$

Substitution of the expression for X_i determined in Equation (3) into this equation gives,

$$\begin{bmatrix} E_t \end{bmatrix}_{\ell x1} = \begin{bmatrix} B_{tj} \end{bmatrix}_{\ell xn} \begin{bmatrix} I - S_{ij} \end{bmatrix}^{-1}_{nxn} \begin{bmatrix} G_{ik} \end{bmatrix}_{nxm} \begin{bmatrix} L_k \end{bmatrix}_{mx1} + \begin{bmatrix} D_{tk} \end{bmatrix}_{\ell xm} \begin{bmatrix} L_k \end{bmatrix}_{mx1} \qquad (6)$$ [18]

Equation (6) provides the value of the total indirect resource requirement, the first term, and total direct resource requirement, the second term. Equation (1) and (4) are in actuality the result of a normalization process that may be derived in the following manner. It is clear that the expressions $\sum_{j=1}^{n} \overline{SX}_{ij}$ and $\sum_{j=1}^{n} S_{ij}\ X_j$ are two equivalent ways of expressing the same matrix-vector product and, therefore, it must be that $\sum_{j=1}^{n} \overline{SX}_{ij} = \sum_{j=1}^{n} S_{ij}\ X_j$. Eliminating the summations in the last expression and equating the products, term by term, yields $\overline{SX}_{ij} = S_{ij}\ X_j$. From this result, it follows that $S_{ij} = (\overline{SX}_{ij}/X_j)$; in similar fashion, $G_{ik} = (\overline{GL}_{ik}/L_k)$, $B_{tj} = (\overline{BX}_{tj}/X_j)$, and $D_{tk} = (\overline{DL}_{tk}/L_k)$. A review of Equations (3) and (6) and Table I shows that it is these expressions that are used in the calculation of total activity input requirements and subsequent computation of total resource requirements. Figure 4 portrays the condition of the tableau after these calculations. At this stage of solution, the individual entries in the tableau have been reduced to the unit level

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[18] The reader is reminded that x_j is simpl[illegible] the transpose of x_i.

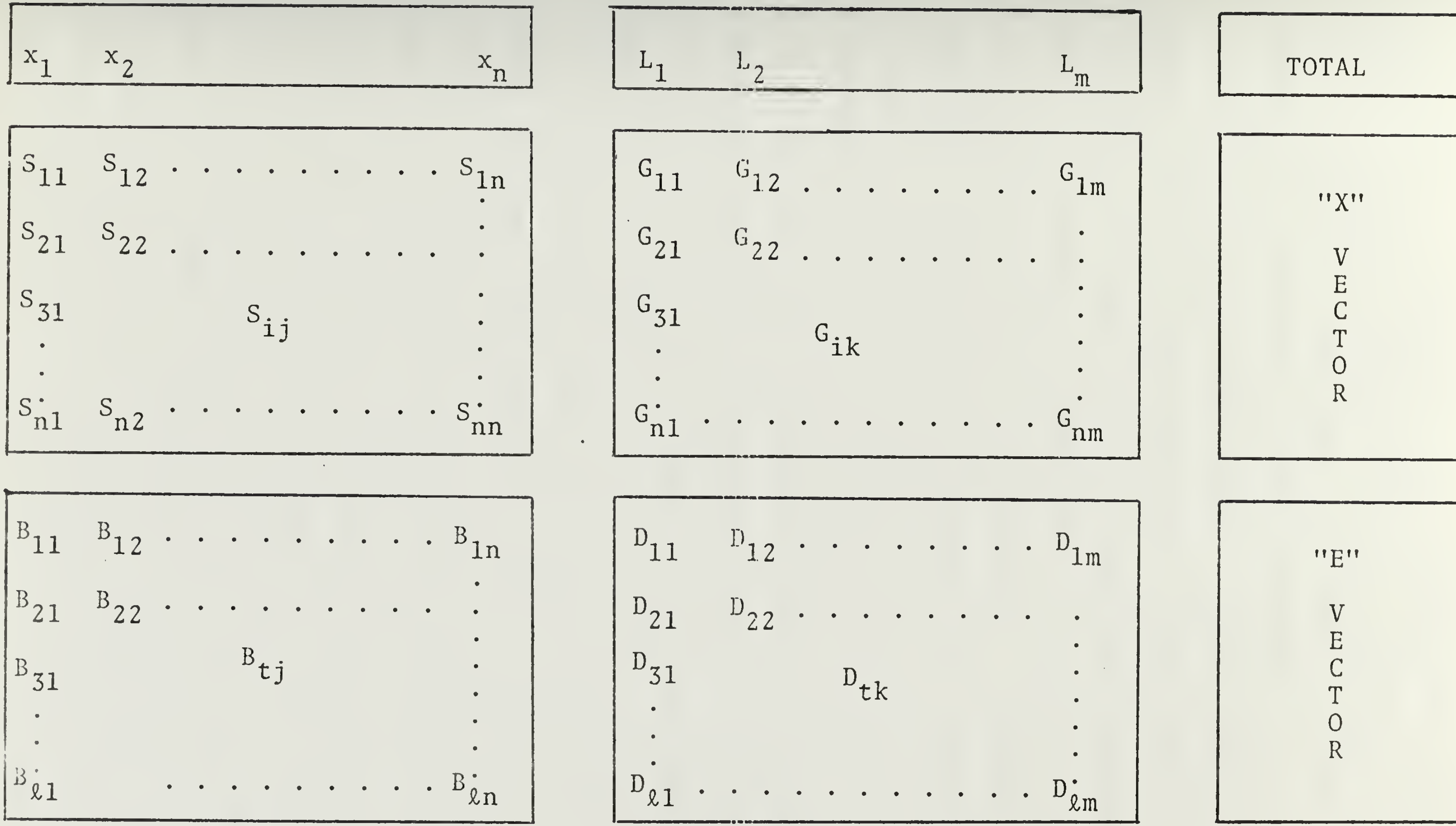

Figure 4. The Normalized Tableau.

of activity, or normalized, for a given level and mix. From Figure 3 it can be observed that $X_i = \sum_{j=1}^{n} \overline{SX}_{ij} + \sum_{k=1}^{m} \overline{GL}_{ik}$ and $E_t = \sum_{j=1}^{n} \overline{BX}_{tj} + \sum_{k=1}^{m} \overline{DL}_{tk}$. Substitution of the normalized results just determined produces Equations (1) and (4).

The remainder of this Chapter will be devoted to a discussion relating to the application of the general model to the school and comments on the assumptions relating to the model. The structure of the school is actually contained in the matrix of Section I of the model. This part of the model must contain all those elements that contribute either directly or indirectly to the production of output if the interrelations among the activities are to give a true picture of how the school functions. The only constraint on the detail presented in this Section of the model is the level to which the input elements may be meaningfully disaggregated. Theoretically, at least, it would be possible to list every person associated with the school as a separate input. However, this author is of the opinion that the expenses incurred in collecting and maintaining data such as this would make such an approach of questionable value. For the most part the inputs of the educational institution listed in this Section of the model will be identified easily. They will include such activities as libraries, computer facilities, academic departments, medical facilities, administrative activities and on and on. These input activities should be found listed as individual entries in the school's budget. The first assumption relating to the input-output

model requires that there be a single output measure for each of the activities listed in the model. Hence, there must be a single output measure for each of the school activities contained in Section I of the model. These output measures provide the data for the model; they are the initial $\overline{SX}_{ij}$ entries listed in the tableau depicted in Figure 3. It is not uncommon to find that a school has not defined activity output measures. It would be nice to have, for example, the number of pieces of paper processed by an administrative department for itself and each of the remaining school activities or to know the number of issues, search requests, or orders processed by the library for each element of the school. In the absence of defined output measures it is always possible (also often necessary) to resort to the use of manhour data available in budget or other management related reports. If the manhour data is available only as an activity total then the total must be apportioned to the remaining school elements. This may be done, for example, by using such a procedure as spreading the total time expended by a payroll section based upon the percentage of total employment for each school activity. A different but equally useful alternative is the identification and use of proxy measures for the entries. This procedure is often necessary when it is impossible to produce or gather any data that can be used to represent realistically the amount of input an activity provides itself and all other input elements. The proxy measures are usually obtained from a

budgetary or other management-type report. An example might be the decision to use the number of faculty members as the initial input coefficients for library or computer facility inputs to other school activities.[19] Section I of the model portrays the explicit and the implicit decision rules relating to the school. The explicit rules are displayed in such ways as faculty to staff ratio or faculty to graduate research assistant ratio being set by the dean or academic council, faculty degree levels necessary to maintain accreditation, or desired faculty to student ratios. Implicit decision rules relate primarily to internal operations. Examples include the provision of secretarial support, operation of duplicating or printing facilities, or janitorial and custodial support. In summary, it may be stated that Section I of the model (with the addition of a student body) represents a microcosm of the school.

The output section of the tableau, Section II of Figure 3, can be used to measure any form of end product desired. These output measures might include such things as numbers of faculty classroom contact hours for students in a given area of study, faculty research effort in terms of published articles, or time spent doing research work. They

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[19] The use of proxy input coefficients has been used in Department of Defense research for construction of some of their models. This approach has provided valuable information to the model builder. Results of initial tests using this approach have been encouraging.

may also be of the form of a value-added concept in terms of measuring the contribution of education to economic growth. In this Section, as in the first, the level of aggregation could span the entire spectrum from listing each student, faculty member, and all other school-connected outputs through a listing by primary courses of study and interest to a gross listing by degree area and faculty department affiliation. Student output, for example, might be measured on an individual basis, by specialty within a given level (B.S. in Education), or just by degree level (B.S., M.S., or Ph.D.). Research might be categorized on an individual basis, by research areas of interest, or by academic department area. Once again it seems that the practicality of measurement and data collection will require aggregation to the level of major courses of study for students and primary area of research interest for research conducted by faculty members.

The remaining two Sections, III and IV, of the model will be discussed together as the level of aggregation for them must be determined from consideration of the detail expressed in Sections I and II. As expressed in Equation (6) the row sums of Sections III and IV comprise the entire school's budget. For this reason the categorization of assets, monetary expenditures, and personnel should reflect the level of control built into the budget. The level of aggregation here will therefore reflect the decision maker's perception of the amount of control that

he must exercise over the school. Again explicit decision rules may be indicated by the development of ratios or minimum and maximum levels that determine the value of any tabular entry in these Sections.[20]

Having completed the discussion relating to the application of the model to the school, the assumptions relating to the input-output model will now be discussed. The first assumption relating to a single output for each activity imposes no conceptual problems. Those activities that in actuality do produce more than one output can be handled by either forming a vector of outputs which would be a listing of all outputs for all activities and therefore primarily composed of zeros for any one activity, or by the disaggregation of an activity into multiple activities each of which has a single output. The second assumption relating to the existence of constant returns to scale, as previously mentioned, means that if all inputs are doubled, tripled, or increased k-fold then the outputs are increased the same respective amount. The final assumption relating to the constancy of the input-output coefficients has been the subject of major criticism of the model. A

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[20] This may be accomplished by establishing the number of professors of a given rank for an academic department or determination of a faculty to student ratio, and then given a student level the faculty size is determined. This last example is drastically simplified as tenure, experience, and other similar items must be considered also in any hiring policy for a school.

particularly good discussion relating to the constancy of the coefficients is provided by Hatanaka.[21] This author believes that certain portions of Hatanaka's criticism is meant to be directed at a causal-explanation prediction model. In view of the current state-of-the-art of input-output cost models care should be exercised to consider Hatanaka's comments in relation to the pure prediction class of models.

Hatanaka's first area of criticism relates to weaknesses resulting from ignoring certain factors considered in production theory. He specifically categorizes shortcomings as being the result of ignoring or overlooking one or more of five basic factors of production. First, the model ignores the occurrence of price substitution. Price substitution, however, is a very real phenomenon and choice among alternative inputs is very dependent on relative input prices. Second, no accounting is made for economies or diseconomies of scale for inputs which are consumed by output elements. Third, certain factors of production which are outside the model are ignored. This criticism results from a failure of the model to account for such things as capital stock or depletion of natural resources. Fourth, the model fails to account for joint production. This shortcoming results from the establishment of a single output

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[21] Hatanaka, Michio, The Workability of Input-Output Analysis, p. 47-60, Fachverlag Fur Wirtschaftstheorie Und Okonometrie Ludwigshafen Am Rhein, 1960.

measure for each of the model's input elements. An example of this joint production is the medical intern or resident. As a doctor this man is himself an output of the system, yet as an intern or resident he contributes to the system's contribution to society or the community. Fifth, and last, the model fails to account for technological progress during the time-frame under consideration. This relates to the inability of the school model to account for or include innovative changes that relate to the variables and data used in construction of the model.

A second area of criticism relates to weaknesses that may occur as a result of the manner in which the model is constructed. Difficulties may arise as a result of the manner in which activities are defined and can be affected by the times when the data are measured. The result, as Hatanaka points out, can be that the input-output coefficients may change as a result of the manner in which the model is aggregated, or as a result of a change in the mixture of old and new production procedures. To visualize the effect of the manner of aggregation assume that there are in fact two constant coefficients. If these coefficients are added during aggregation of the model there is no longer any reason to assume that the constancy of the individual coefficients has been maintained. The mixing of old and new production procedures can be clearly illustrated by considering an industrial firm that is faced with a reduction of demand for its product. Assume, for the

purposes of this example, that this industry does in fact operate an old and a new production process in producing its output. When the industry's sales drop, it will undoubtedly cut back on production by the old process. When business improves, and sales begin to increase again, the firm will probably choose to expand by increasing its new process rather than restarting the old process. The end result will be that the relative proportions of old and new production lines have changed and hence the input-output coefficients have changed.[22] A final problem may result from the way in which the model is built. This relates to what Hatanaka calls the dilemma of aggregation. This problem relates to the two-sided nature of input-output, or the fact that we must study the activity's providing role of furnishing its output as input to an intermediate or final output activity and also the activity's consuming role of using supplied inputs in the production of output. Difficulties may occur in physically classifying inputs and outputs as the input-output structure requires that an activity's consuming role classification be used in the determination of its classification for providing its input to other activities. This problem area should not present major problems in applying

[22] It really does not matter which process is cut back as the combinations available to the firm when business picks up can still lead to a change in relative rates, and hence a change in the coefficient values.

input-output procedures for the construction of a school cost model. However, consider for a moment an industrial application of input-output modelling at the national economy level. The magnitude of the problem using this classification scheme to categorize all the industries of that nation is apparent.

In closing this Chapter, the problem of validation of a model such as this will be discussed briefly. There are essentially two approaches that may be taken to determine the ability of the model to provide accurate forecasts. The difference in these approaches is related only to the time period for which the predictions are made. One method of prediction is to construct the model now and predict subsequent budgets. The other possibility is to use historical data available in the school to build the model and predict a budget for which a realized value already exists. In other words if budgetary data is available in a useable form from 1950 to 1970, use 1950's data to build the model and then predict 1951's budget, 1951's to predict 1952's, and so on until 1969's data is used to predict 1970's budget. It would seem to this author that this approach is more promising than the predict and wait alternative. Once the predictions have been made, irregardless of which approach is used, and actual budget realizations are available, the predicted and realized value are compared and an error frequency distribution is constructed for the prediction errors.[23] Once this has

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[23]Jones, op. cit.

been accomplished it is then possible to repeat the same procedure using any existing model the school has been using or any other models of interest. Comparison of the various results will then provide information relative to the accuracy of this model in comparison to the others tested.

IV. APPLICATION OF THE INPUT-OUTPUT COST MODEL TO THE NAVAL POSTGRADUATE SCHOOL

In order to demonstrate the manner in which a school may be cast into the input-output framework and to show some of its uses the general model will be applied to the Naval Postgraduate School. To simplify the understanding of the application of this model, or for that fact any other model, to a school it is necessary to become familiar with the basic operation of the school.

The Naval Postgraduate School primarily confers degrees at the Master of Science level. The School also confers baccalaureate, engineer, and doctoral-level degrees. Table II presents a listing of the various curricula offered by the School. For the most part student input occurs twice a year and the various Master's level programs are of approximately eight quarters, or two years, duration. The instruction presented by the School comes from a faculty organized into the academic departments listed in Table III. The faculty is required also to support and maintain an active research program in addition to its teaching responsibility. The faculty is composed principally of civilian professors with some military instructor supplementation. Student organization and control is provided through the military-directed program elements listed in Table IV. This program organization assists in the student's academic development as well as accomplishing necessary academic and military related record-keeping

Table II. Curricula at the Naval Postgraduate School*

Curriculum	Curriculum Number
Advanced Science	380
Aeronautical Engineering	610
Baccalaureate	461
Communications Engineering	600
Engineering Electronics	590
Engineering Electronics (Special)	472
Communications Management	620
Engineering Science	460
Meteorology	371
Advanced Meteorology	372
Oceanography	440
Computer Science	368
Computer Systems Management	367
Management	814 and 817
Mathematics	430
Naval Engineering	570
Operations Research/Systems Analysis	360
Nuclear Engineering (Effects)	521
Underwater Physics Systems	535
Ordnance Systems Engineering	530

*Extracted from the Naval Postgraduate School Catalogue for 1970-1972.

Table III. Academic Departments at the Naval Postgraduate School*

Meteorology Department
Electrical Engineering Department
Mathematics Department
Material Science and Chemistry Department
Operations Analysis Department**
Government and Humanities Department
Aeronautics Department
Mechanical Engineering Department
Physics Department
Business Administration and Economics Department**
Oceanography Department

* Extracted from the Naval Postgraduate School Catalogue for 1970-1972.

**These two academic departments were merged in 1971 to form the Operations Research and Administrative Sciences Department.

Table IV. Program Elements at the Naval Postgraduate School*

Aeronautical Engineering Programs
Electronics and Communications Engineering Programs
Ordnance Engineering Programs
Naval Engineering Programs
Environmental Sciences Programs
Management and Computer Science Programs
Engineering Science Programs
Baccalaureate Programs
Operations Analysis Programs

*Extracted from the Naval Postgraduate School Catalogue for 1970-1972.

functions. Academic assistance is provided through the program office serving as an interface between the student and the academic departments in establishing and coordinating elective sequences, determining curricula requirements, and in other related activities. The curricula offered by the School are designed to fulfill specific requirements within the Navy and hence subject to review and approval by agencies external to the School.

Having presented this brief description of the operation of the Naval Postgraduate School, it is now possible to cast the School into the input-output framework developed in Chapter III. For the purposes of this thesis the outputs of the School are considered to be graduates and faculty research.[24] A graduate, rather than a student, was chosen as the output measure because the graduate was initially considered to be more representative of an output measure. The use of a graduate rather than a student also simplified the data collection effort. It should be noted that with this particular output measure it would be possible for an individual to attend the school, place demands on the system, and never be counted as output. The chance of this happening at the Postgraduate School is not too great, however, due to the awarding of Diplomas of Completion to those individuals not qualifying for a

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[24] Graduates were considered to include those individuals who received "Diplomas of Completion" as well as actual academic degrees. This was done as these individuals did in fact complete a full course of instruction.

degree. Further discussion of output measures and their impact on phase I and phase II of the total research effort will be discussed later. Inputs to these output elements in addition to faculty instruction hours and faculty research effort hours consist of manhours of support from such activities as the library, computer center, and other similar School elements. The structure of the School, Section I of the model developed in Chapter III, was taken to be the configuration established in conjunction with the School's Resource Management System.[25] This system has identified 44 data collection points or cost centers and the procedures for the collection of data related to resource utilization and expenditures. The input data available from this reporting scheme is limited to the total number of manhours and total amount of expenditures for each of the data collection points. Provisions have been included for the use of work unit measures in this accounting scheme, however, only a few such measures have been identified. These 44 cost centers, with minor modifications, have been taken to be the framework for the School, or Section I of the model. The fiscal year-end reports prepared in conjunction with the Resource Management System have provided total manhour and dollar expenditures for each of the cost centers in the school. The

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[25] Naval Postgraduate School Instruction 7110.1C, Subject: Management of Resources, 14 October 1970.

42 elements that were used in the final construction of the square matrix of Section I of the model are listed in Table V.

The summary data contained in the RMS reports for fiscal year 1970 was used to construct the tabular model for the Postgraduate School. Since this data consisted of a total number of manhours and dollar expenditures for each element of the school, the M_i and the O_i respectively in terms of Table I and Figure 3, it was necessary to develop procedures to construct the individual row entries which produced the individual M_i's and O_i's. This was necessary for the solution of equations (3) and (5) in the process of using the model to predict future requirements. If the individual cost centers maintained information in such a way that the amount of support provided to itself and other school elements would have been available, these procedures would not have been necessary. To accomplish the apportioning of each row sum into the individual $\overline{SX}_{ij}$ and $\overline{BX}_{tj}$ components, which represent respectively the level of the i^{th} cost center's input to the j^{th} cost center and the amount of resource t required for the j^{th} graduate, seven basic population aggregations for the School were identified. These groupings were selected by looking at the work accomplished by each of the 42 cost centers in the School and identifying for each cost center a population element that primarily consumed its service. For example, the cost center concerned with civilian personnel actions provided its

Table V. The Support Establishment of the Naval Postgraduate School*

Aeronautical Engineering Programs
Electronics and Communications Engineering Programs
Ordnance Engineering Programs
Naval Engineering Programs
Environmental Sciences Programs
Management and Computer Science Programs
Engineering Science Programs
Baccalaureate Programs
Operations Analysis Programs
Meteorology Department
Electrical Engineering Department
Mathematics Department
Material Science and Chemistry Department
Operations Analysis Department
Government and Humanities Department
Aeronautics Department
Mechanical Engineering Department
Physics Department
Business Administration and Economics Department
Oceanography Department
Academic Dean/Deputy Superintendent for Programs
Dean of Curricula
Library Science Department
Computer Center
Machine Facility
Educational Media Department
Comptroller
Civilian Personnel Officer
Supply Department
Public Works Officer
Superintendent's Aide
Director Administrative Department
Security and Boats Officer
Aviation Safety Programs
Dental Officer
First Lieutenant
BOQ and Closed Mess Officer
Staff Judge Advocate
Chaplains
Public Affairs Officer
Recreation Officer
Textbook Library

*Extracted from NPS Inst 7110.1C; Management of Resources; dated 19 October 1970.

service to civilian employees of the School and not to military personnel assigned to the School. The seven groupings finally chosen were School civilian and military staff plus graduate population, civilian population, military staff plus graduate population, faculty plus graduate population, program area population, faculty population, and total expenditures. A complete break out of the 42 School cost centers listed by the population grouping used to apportion their total manhours is shown in Table VI. Having identified the population groupings, a representative individual row entry for a primary civilian function would be calculated in the following manner.

$$\overline{SX}_{ij} = (M_i)\left(\frac{\text{Civilian population for school element j}}{\text{total civilian population}}\right).$$

Similar calculations were made for each of the other six population groupings until each row sum had been apportioned into its individual elements. These individual entries were then used to construct the square matrix of Section I of the model. The entries further serve to identify the inputs, outputs, and related flows that exist in the school since they portray the i^{th} activity's input to the j^{th} activity. The reader is reminded that these entries in the model are all in terms of manhours.

The data for Section II, the output elements, of the model was gathered from two different sources. First, the Resource Management System reports were used to obtain the number of manhours and dollar expenditures that each academic

Table VI. Apportioning Scheme for Cost Center Manhour Totals*

School civilian and military staff plus graduate population	Civilian Population
Security and Boats Officer Public Works Officer	Civilian Personnel Officer
Military Staff plus graduate population	**Program Population**
Dental Officer Director of Administrative Department First Lieutenant BOQ and Closed Mess Officer Staff Judge Advocate Chaplains Public Affairs Officer Recreation Officer Superintendent's Aide	The 10 School Program Areas Machine Facility Text Book Library
Faculty plus graduate population	**Faculty Population**
Library Science Department	Dean of Curricula
Expenditures	
Comptroller Supply Department	

* Data by cost center had been maintained by the Educational Media Department and the Computer Center and so no population apportioning of their manhour totals was required. The eleven academic departments and the Aviation Safety Program manhours were assigned on a demand basis as explained in the text.

department devoted to research. Second, records available within the school were reviewed and the number of graduates by degree area for fiscal year 1970 were determined. With these figures, it was then possible to consult the Naval Postgraduate School Catalogue for 1970-1972 and to determine the total teaching requirement in terms of instructional hours for each of the academic departments for the 1970 graduation level and mix. This was possible since the Catalogue assigns responsibility for each course offered by the School to a specific academic department. The Catalogue further identifies the curricula requirements necessary to obtain a given degree. In this present scheme elective hours were assigned arbitrarily to the graduate's degree area, e.g. course hours for electives for graduates receiving physics degrees were added to the instruction demands placed on the Physics Department, for degrees in operations analysis to the Operations Analysis Department, and so on. Also included in this Section of the model were inputs, in manhours, from those other School activities whose functions had been related to graduate population, military population, or program area population.

The data for the remainder of the model, Sections III and IV, was obtained from the Resource Management System reports or from other internal elements of the School. Expenditure data for each of the School elements and for research effort was obtained from the Resource Management System reports. The authorized number and grade structure

of civilian and military staff members was obtained from the School's manpower listing [13]. This data provided the individual entries for the $\overline{BX}_{tj}$ and $\overline{DL}_{tk}$, the resource levels required respectively to produce the j^{th} or k^{th} output, components in the model. With this brief explanation of the elements of the Naval Postgraduate School and the description of the functional-based population elements used to apportion total outputs and total expenditures it is possible to construct a specific model for the School in the framework of Figure 3. The actual framework used for the School with sample entries is shown in Figure 5.

In applying the input-output model to the Naval Postgraduate School it was desired to investigate the capability of the model to accomplish four possible actions. First, it was desired to determine expenditure forecasts for a given level and mix of graduates. Second, it was desired to examine the effect of changes to the School elements or output areas. Caution must be exercised in dealing with changes in the elements of the model. Linear approximations within the vicinity of an operational point are all that can be expected to hold. This process may be viewed also as developing the Taylors' series expansion about a particular point and neglecting all but the linear terms. Third, it was desired to portray the interrelationships of School and output elements by identification of inputs, outputs, and related flows. Finally, it was desired to be able to compute the incremental cost of an individual graduate or

		OA Department	* *	Security Boats	M.S. O.A.	Research	Total
OA Department		20590			26901	23912	81911
* *							
Security Boats		1450		1128	3230		59039
Civilian Salary	($)	551,697				*	
Military Pay	($)	23,762				*	
Other	($)	16,812				*	
Professor	(#)	4					
Associate Professor	(#)	14					
Assistant Professor	(#)	17					
Military Instructor	(#)	3					
Military Officer	(#)			1			
Military EM	(#)			17			
Civilian	(#)	6		16			
Total	(#)	44		34			

* Denotes that these figures represent the total dollar expenditure in each category that is devoted to research.

** Each element listed in Table V appears as a row and a column. Row entries are in manhours.

Figure 5. Sample Tabular Entries for the Naval Postgraduate School.

to analyze the expenditure changes that occur as a result of adding or deleting a graduate output area.

In discussing the output generated in exercising the model only percentages and trends of expenditures will be presented since this simplifies presentation of the results. Full information relating to the actual numbers obtained is available at the Postgraduate School. Table VII contains percentage results of using the fiscal year 1970 data base to backcast fiscal year 1969 expenditures and to forecast fiscal year 1971 expenditures. The values expressed in the Table are percentage differences, in dollar terms, between the forecast and realized expenditure, with the realized expenditure used as the base.[26] The column heading, faculty preparation factor, concerns the amount of time allowed in a given curriculum for faculty instructional efforts. It was decided during the initial design of the School model to start with a factor of two (this means that a faculty member was allowed two-hours preparation time for each instructional hour) and to vary the factor from this initial value. This factor was introduced since clearly a faculty member devotes more time to a graduate than just the three or four classroom instruction hours of a particular course. Time expended by the faculty member for lecture preparation, test preparation

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[26]The operation used to calculate the Table entries can be expressed as follows:

$$\text{Percentage difference} = \frac{\text{Forecast(\$)} - \text{Realization(\$)}}{\text{Realization (\$)}}$$

Table VII. Model Percentage Error Predictions*

	FACULTY PREPARATION FACTOR		
	2.0	2.5	3.0
FY 1969 BACKCAST	+18.9%	+8.6%	+0.8%
FY 1971 FORECAST	+ 7.8%	-1.6%	-8.6%

*Note: A plus percentage factor indicates that the dollar forecast was greater than the dollar realization while a negative factor was less.

and grading, thesis assistance, and other similar activities should be attributed to graduate output. In order to account for this the model was exercised with those factors contained in Table VII. This varying of the faculty preparation factor merely amounts to a parametric analysis in an attempt to improve the prediction ability of the model. This is permissible since the model has been viewed as a pure prediction model. As the reader can observe, the arithmetic value of the sum of the two error predictions decreases greatly between 2.0 and 2.5 and increases slightly between 2.5 and 3.0. When the prediction errors are considered as a vector of the absolute values the 2.5 factor is associated with a vector prediction error which dominates the 2.0 factor. The 3.0 factor dominates the 2.0 factor but not the 2.5 factor.

Two observations relative to the results displayed in Table VII are appropriate. First, there were some changes in the manhour and expenditure reporting scheme of the Resource Management System between fiscal years 1969 and 1970. These changes resulted in the transfer of responsibility for reporting manhours and expenditures among some of the cost centers. The result of a change such as this will reduce the accuracy of forecasts unless the coefficients for the school elements involved are modified to reflect these changes prior to computation of a forecast. The type of change referred to here would involve, for example, a transfer of the reporting responsibility for personnel between

any two or more elements of the school. These changes were not accounted for and hence affect the fiscal year 1969 backcast from the fiscal year 1970 data base. Second, there was a failure to allow for civilian and military pay raises in the original formulation of the model. This discrepancy is easily overcome as shown in the final School model depicted in Figure 5. By listing military and civilian pay as individual vector entries in Section III of the model it is possible to include the percentage raise for each group as a constant multiplier in the appropriate rows. The current reporting scheme for the School separates military and civilian pay and so this modification is not a problem.

The incremental cost of one more graduate predicted by the model may be interpreted in alternative ways. It is not inappropriate to interpret that the school is in fact in a true state of long-run equilibrium. In this case the costs predicted by the model for the addition of one more graduate should be representative of the long-run costs associated with steady-state conditions. It is doubtful, however, that such long-run equilibrium conditions existed during the period investigated. This is exemplified by the student/graduate ratio for the years investigated. These values were:[27]

FY 1969	1412/633
FY 1970	1757/785
FY 1971	1867/908

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[27]Student totals are as of the fourth quarter in each fiscal year.

Even though the existence of true equilibrium is questionable, it would be inappropriate to go to the complete other extreme and conclude that the School cannot vary any of its resource usages. In the middle ground is the short-run which is considered to be that period in which some but not all resource usages may be varied. The School's staff employment policies and the existence of faculty tenure represent examples of two specific areas where resource commitments for several years are incurred. For example, instructor contracts must be written generally for periods in excess of a year and tenure can preclude, or at least complicate, short notice faculty strength reductions. Thus, even though the input-output model might predict a decreased instructional requirement for a particular academic department it would not usually be practical to reduce the faculty size of that department in the short-run. This phenomenon could lead to the existence of excess instructional resources in such quantities that additional students could be added at no marginal cost. Since the initial development of the input-output model does not account for such resource immobilities, the predicted costs for short-run operation will be biased upwards where there exist excess resources due to long-run commitments.

The use of a student rather than a graduate as the output measure should reduce the size of the incremental cost prediction. This should occur since the addition of these School members will reduce the size of some of the coefficients in Section I of the model. This change would also

seem to be more satisfying conceptually since the School's resources are consumed by all students, not just graduates.

The model did appear, in the author's opinion, to be able to provide a consistent ranking of the different graduate area incremental costs. It would seem to be intuitively consistent to expect the incremental costs of engineering or science graduates to be greater than those of management or business-oriented graduates. This is in fact exactly what the model does predict. Table VIII lists the decreasing incremental cost sequence for a selected group of degree areas at the Postgraduate School. Time constraints did not allow the development of an exhaustive list for all degree areas since each calculation required the manual computation of the output requirements for the additional graduate. For this reason incremental costs for degree areas that were considered by the author to be representative of the different types of instruction provided by the school were computed, e.g. engineering, science, and management science. The arithmetic ratio listed with each degree area is a ratio of predicted dollars using the Master of Science in Management (with a 2.5 faculty preparation factor) incremental cost as the base cost. Specifically,

$$\text{Arithmetic Ratio} = \frac{\text{Degree Incremental Cost (\$)}}{\text{MS, Mgt Degree Incremental Cost(\$)}}$$

In the author's opinion the current predicted incremental cost of an additional graduate using the model appears to be too large. This conclusion has been reached after discussions with school officials, comparison of the predicted

Table VIII. Decreasing Incremental Cost Sequence for Selected Degree Areas at the Naval Postgraduate School

Degree Area	Arithmetic Ratio
Master of Science, Electrical Engineering*	2.37
Master of Science, Oceanography	1.64
Master of Science, Operations Analysis	1.37
Master of Science, Management	1.00

*This is for a graduate of curriculum number 570 in the Naval Engineering Program area. This incremental cost was the largest of the degree areas investigated.

incremental costs to costs at other schools, from analysis of previous studies and reports relating to average costs prepared by School officials, and the discussion contained in the preceeding paragraphs concerning short-run costs and resource mobility.

The model was not exercised to demonstrate its use in examining the effect of changes in School elements. It is not difficult to envision, however, how this problem would be handled. In the case of an addition or deletion, the appropriate change is made to either the matrix of Section I of the model or to the output listing of Section II of the model. Once this has been done the new values of the M_i's or N_i's, the total intermediate and final requirements respectively for the i^{th} cost center's input, are computed and the prediction procedure is accomplished in the usual manner. If an internal change occurs, the appropriate $\overline{SX}_{ij}$ or $\overline{GL}_{ik}$, the i^{th} cost center's input level to another cost center or a graduate respectively, values are corrected and once again the prediction is computed in the normal manner.

The inputs, outputs, and related flows that go together to comprise the School are depicted ultimately in the complete tableau of the School. Figure 5 portrays the general framework of the model and is illustrative of the type of information available in the tabular formulation. Examination of the individual rows and columns of this tableau specifically identifies the inputs and outputs and, more importantly, the interrelated flows. For example, the flow

involving the Engineering Science Program is depicted in Figure 6. This particular program has no direct output since its primary purpose is to provide courses for individuals going on to studies in other fields of science or engineering.

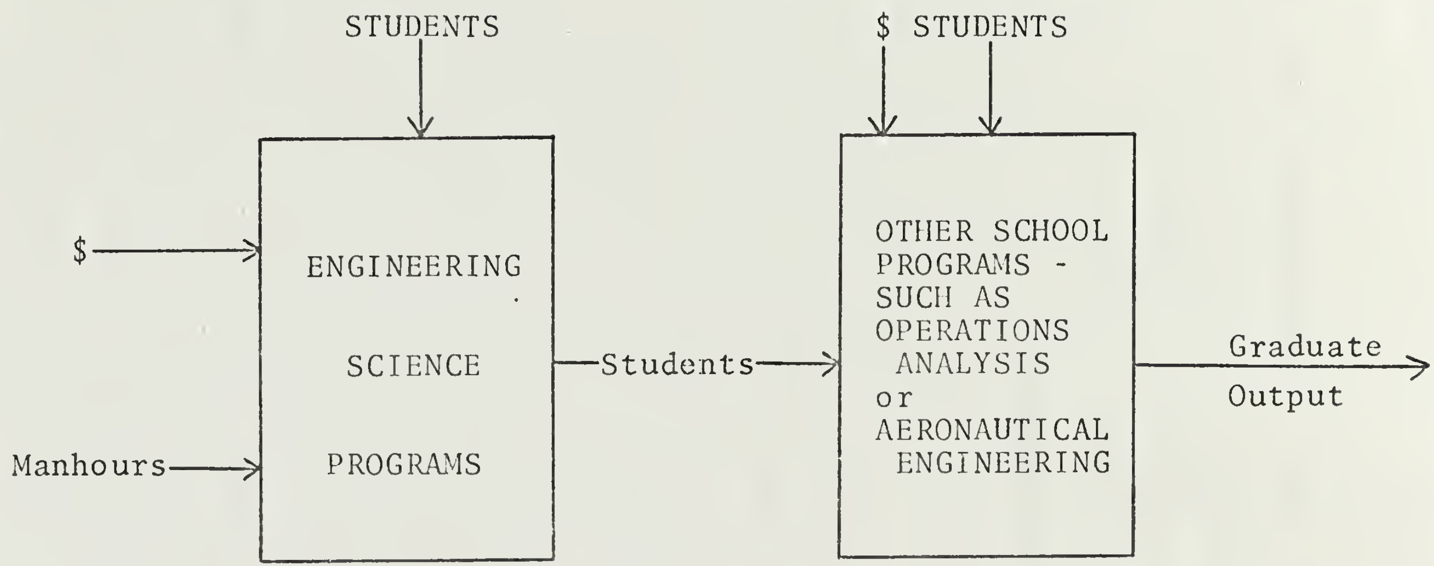

Figure 6. Sample Interrelated Naval Postgraduate School Flows.

V. SUMMARY AND AREAS FOR FURTHER STUDY

In summary, it has been shown that input-output procedures can be applied to the construction of a cost model for an educational institution. It has been shown that the modification of the classical Leontieff model into the tableau form of the "Electric" Five-Year Defense Plan can provide a model for the school that is limited in detail only by the level for which meaningful data is available. The relatively simplistic mathematics and supporting tableaus for the model were developed.

The procedures developed for the general case were applied to the Naval Postgraduate School and the model was exercised to determine the ability of the model to forecast budgets for a given level and mix of graduates; to examine the effects of changes to School elements or output areas; to portray the interrelationships that exist both internally in the School and externally in the output of the School; and to predict incremental costs for a given output area. The results in each of these areas were encouraging.

As a result of the research conducted for the preparation of this thesis there seems to be, at least to this author, three areas in which the use of input-output cost models can be of immediate general use. First, the casting of an organization into the input-output framework and the resulting explicit identification of the inputs, outputs, and related flows can assist the manager in better understanding the operation of his organization. Second, if used

in a manner as discussed in Chapter IV on the application of the model to the Postgraduate School the predictions can be used at least to indicate the existence of excess or deficit resources. In this case the manager can then investigate those areas where excesses or deficits are predicted in more detail to determine if any immediate action is required. For example, it may turn out that an excess resource does exist. With this knowledge it may be possible to reduce costs in some other area of the organization by making better use of the existing excess resource. Finally, it should be possible to begin to build actual organizational input-output models using procedures similar to those described in this thesis. Once the models have been constructed it will be possible to initiate validation tests and upon successful validation to begin to use them in ways similar to those previously discussed.

There are several areas which seem to be suitable for further study and research effort in relation to the use of input-output procedures in cost modelling. The comments that follow will refer specifically to the model of the Naval Postgraduate School since it is the model to be used in subsequent efforts. First, a better method of determining the amount of output that each element of the School provides as input for itself, other School elements, and output elements should be developed. This will mean, essentially, the definition of output measures for each element of the School and the maintenance of data relative to the amount of support provided to another School element or

output area. Efforts devoted to this area should prove most fruitful, in this author's opinion, as this will allow a more accurate identification of the School interrelationships than did the seven populations used in this thesis. This more accurate identification of support requirements should also help to refine and improve the budget forecasts and incremental costs predicted by the model.

A second area for further study relates to the output measures for the School. This area is related closely to the problem just discussed, and could be addressed either independently or concurrently with the first area. The use of a graduate as an output measure leads to fluctuations in resource demands placed on the School that in fact do not occur. For the most part, enrollment at the School does not change drastically from one year to the next. In a small school (such as the Naval Postgraduate School with its enrollment of approximately 1800 students) the use of graduates as the output measure can lead to unrealistic predictions. A much more meaningful output measure would seem to be total students aggregated on the basis of degree or program area. This particular output measure was not used in this thesis since it was believed that the output measure chosen for initial use in phase I of the total research effort would not have too drastic an effect on the results obtained. As mentioned earlier, the use of a graduate also served to expedite the construction of the first empirical model due to the fact that data was more readily

available for a graduate than a student. It is interesting to note again at this point the results displayed in Table VII. As mentioned previously in Chapter IV, the student to graduate ratio for the School is slightly larger than two to one. The reader will observe from Table VII that the closest prediction for the fiscal year 1971 forecast lies somewhere between a faculty prediction factor of 2.0 and 2.5. This faculty prediction factor has the same effect on the forecast as increasing the number of graduates by the amount of the factor. This results since either action has only a multiplicative effect on Section II of the model. The closeness of the predicted and realized values with this 2.0 and 2.5 factor included would seem to reinforce the value of conducting additional research with the output measure taken as the student. As has been indicated, the requirement for further research in these first two areas has been discovered as a result of the current author's research and thus relate to phase I of the overall research strategy. For the most part, they have been discovered as a result of the construction and empirical use of the actual model for the Naval Postgraduate School. It is possible that these two refinements of the model will terminate phase I of the research effort.

Finally, it would be desirable for the model to be able to answer or to address problems relating to segmentation, the grouping of students into an instructional group. Further efforts along this line will require redefinition of Section

II of the model in such a manner as to account for actually placing the students into a planned class or section structure so that instructional resource requirements can be determined on a group rather than individual basis. A possible solution to the segmentation issue might result from identification of the output measure for a time period less than a full academic year. In this case the output measure would become, for the Naval Postgraduate School, the number of students in a given degree area for a particular quarter of the school year. The advantage of this approach is that the model would depict the current status of the School's resources in a time frame that coincides with the normal scheduling function. With this approach the model would be utilized four times recursively to generate an annual prediction. These procedures should not be confused with a separate quarterly scheduling algorithm. What is desired is a method of linking projected output through the model to the School's available resources so that planning factors for segmentation might possibly be developed. These factors would then provide information about the size of instructional groups for different output areas. Efforts in this area, as mentioned in Chapter I, will be conducted during phase II. Progress in this particular area will be extremely valuable to the School as it will assist in the explicit identification and determination of instructor requirements. Solution of this problem will aid the School in answering important questions relating to xcess or deficit instructor resources.

APPENDIX A: EMPIRICAL DATA FOR NAVAL POSTGRADUATE SCHOOL MODEL

This Appendix consists of two tables. Table I contains the $[I-S_{ij}]$ square matrix for the Naval Postgraduate School used for computation of the fiscal year 1971 forecast. This data is fiscal year 1970 historical data. This matrix allows for a 2.5 faculty preparation factor in Section II of the complete model. Table II contains two vectors. These vectors are respectively the product of the $[G_{ik}]$ matrix (output requirements) and L_k vector (output level) and the normalized $[B_{tj}]$ matrix which as used in preliminary runs was a single vector containing the normalized FY 1970 total dollar expenditures of each of the cost centers. For each of the vectors, row one contains entries 1-6, row 2 entries 7-12, row 3 entries 13-18, row 4 entries 19-24, row 5 entries 25-30, row 6 entries 31-36, and row 7 entries 37-42. These 42 total entries are for the 42 cost centers contained in Section I of the model.

TABLE I. $[I-S_{ij}]$ TABULAR ENTRIES FOR THE NAVAL POSTGRADUATE SCHOOL

ROW(1)

1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(2)

0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(3)

0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(4)

0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(5)

0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(6)

0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(7)

-0.048	-0.119	-0.062	-0.129	-0.100	-0.269	1.000	0.0
-0.125	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(8)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(9)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(10)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.273	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(11)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.276	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(12)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.705	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(13)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.262	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(14)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.681	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(15)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.618	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(16)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.334
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(17)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.377	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(18)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.421	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

```
                                    ROW(19)
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.746   0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(20)
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.479   0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(21)
 -0.212  -0.524  -0.275  -0.569  -0.444  -1.189   0.0    -0.510
 -0.553   0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     1.000   0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(22)
  0.0    -0.027   0.0     0.0     0.0     0.0     0.0     0.0
  0.0    -0.014  -0.012  -0.019  -0.018  -0.021  -0.019  -0.012
 -0.015  -0.019  -0.021  -0.019  -0.002   0.996   0.0    -0.001
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0    -0.023   0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(23)
  0.0    -0.033   0.0     0.0     0.0     0.0     0.0     0.0
  0.0    -0.017  -0.014  -0.023  -0.021  -0.025  -0.023  -0.014
 -0.019  -0.023  -0.025  -0.023  -0.002  -0.005   1.000  -0.001
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0    -0.028   0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(24)
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0    -0.008   0.0     0.0     0.0     0.0
 -0.028  -0.034   0.0    -0.013   0.0     0.0     0.0     0.580
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(25)
 -0.339  -0.840  -0.440  -0.911  -0.711   0.0     0.0     0.0
 -0.886   0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  1.000   0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
  0.0     0.0
                                    ROW(26)
 -0.009  -0.029  -0.008  -0.011  -0.009  -0.019   0.0    -0.013
 -0.006  -0.014  -0.021  -0.029  -0.006  -0.023  -0.020  -0.012
 -0.026  -0.008  -0.084  -0.023  -0.032  -0.126  -0.040  -0.036
  0.0     0.775  -0.005  -0.081  -0.019  -0.007  -0.021  -0.027
 -0.001  -0.104  -0.002  -0.001   0.0    -0.011  -0.087  -0.043
 -0.027   0.0
                                    ROW(27)
 -0.025  -0.024  -0.022  -0.024  -0.024  -0.020  -0.017  -0.021
 -0.031  -0.030  -0.033  -0.033  -0.037  -0.034  -0.030  -0.034
 -0.038  -0.038  -0.034  -0.036  -0.029  -0.020  -0.026  -0.030
 -0.014  -0.022   0.984  -0.020  -0.017  -0.033  -0.017  -0.014
 -0.012  -0.030  -0.016  -0.011  -0.009  -0.027  -0.031  -0.015
 -0.012  -0.033
                                    ROW(28)
 -0.004  -0.003  -0.004  -0.005  -0.004  -0.006  -0.008  -0.006
 -0.006  -0.009  -0.010  -0.008  -0.013  -0.011  -0.012  -0.012
 -0.011  -0.012  -0.009  -0.012  -0.013  -0.015  -0.013  -0.012
 -0.005  -0.011  -0.008   0.989  -0.006  -0.012  -0.001  -0.004
 -0.006  -0.010  -0.001  -0.006   0.0    -0.005  -0.003  -0.005
 -0.001   0.0
```

ROW(29)

-0.054	-0.051	-0.047	-0.051	-0.051	-0.043	-0.038	-0.046
-0.067	-0.066	-0.071	-0.070	-0.081	-0.074	-0.064	-0.074
-0.082	-0.081	-0.074	-0.077	-0.062	-0.042	-0.056	-0.079
-0.031	-0.047	-0.034	-0.043	0.918	-0.073	-0.038	-0.031
-0.027	-0.065	-0.035	-0.024	-0.019	-0.059	-0.068	-0.032
-0.025	-0.072						

ROW(30)

-0.085	-0.180	-0.074	-0.106	-0.079	-0.077	-0.107	-0.165
-0.080	-0.079	-0.079	-0.067	-0.085	-0.079	-0.079	-0.086
-0.073	-0.084	-0.077	-0.088	-0.093	-0.097	-0.084	0.0
-0.090	-0.071	-0.073	-0.071	-0.067	0.916	-0.058	-0.063
-0.085	-0.096	-0.061	-0.055	-0.058	-0.070	-0.072	-0.050
-0.125	-0.059						

ROW(31)

-0.010	-0.028	-0.009	-0.013	-0.009	-0.007	-0.010	-0.022
-0.007	-0.003	-0.003	-0.002	0.0	-0.001	0.0	-0.001
0.0	-0.001	-0.003	-0.002	-0.002	0.0	0.0	0.0
-0.010	0.0	-0.004	0.0	-0.005	-0.001	0.991	-0.007
-0.008	-0.006	-0.010	-0.003	-0.010	-0.006	-0.010	-0.003
-0.021	-0.011						

ROW(32)

-0.040	-0.111	-0.035	-0.050	-0.037	-0.027	-0.038	-0.087
-0.028	-0.013	-0.011	-0.009	0.0	-0.004	0.0	-0.003
0.0	-0.003	-0.012	-0.008	-0.008	0.0	0.0	0.0
-0.038	0.0	-0.014	0.0	-0.018	-0.004	-0.037	0.973
-0.032	-0.022	-0.040	-0.011	-0.041	-0.025	-0.038	-0.012
-0.083	-0.041						

ROW(33)

-0.019	-0.040	-0.017	-0.024	-0.018	-0.017	-0.024	-0.037
-0.018	-0.018	-0.018	-0.015	-0.019	-0.018	-0.018	-0.019
-0.016	-0.019	-0.017	-0.020	-0.021	-0.022	-0.019	-0.018
-0.020	-0.016	-0.016	-0.016	-0.015	-0.019	-0.013	-0.014
0.981	-0.021	-0.014	-0.012	-0.013	-0.016	-0.016	-0.011
-0.028	-0.013						

ROW(34)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.165	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0						

ROW(35)

-0.011	-0.032	-0.010	-0.014	-0.011	-0.008	-0.011	-0.025
-0.008	-0.004	-0.003	-0.002	0.0	-0.001	0.0	-0.001
0.0	-0.001	-0.003	-0.002	-0.002	0.0	0.0	0.0
-0.011	0.0	-0.004	0.0	-0.005	-0.001	-0.011	-0.008
-0.009	-0.006	0.989	-0.003	-0.012	-0.007	-0.011	-0.003
-0.024	-0.012						

ROW(36)

-0.018	-0.049	-0.015	-0.022	-0.016	-0.012	-0.017	-0.039
-0.012	-0.006	-0.005	-0.004	0.0	-0.002	0.0	-0.001
0.0	-0.001	-0.005	-0.003	-0.004	0.0	0.0	0.0
-0.017	0.0	-0.006	0.0	-0.008	-0.002	-0.017	-0.012
-0.014	-0.010	-0.018	0.995	-0.018	-0.011	-0.017	-0.005
-0.037	-0.018						

ROW(37)

-0.040	-0.110	-0.035	-0.049	-0.037	-0.027	-0.038	-0.087
-0.028	-0.013	-0.011	-0.009	0.0	-0.004	0.0	-0.003
0.0	-0.003	-0.012	-0.008	-0.008	0.0	0.0	0.0
-0.038	0.0	-0.014	0.0	-0.018	-0.004	-0.037	-0.026
-0.032	-0.022	-0.040	-0.011	0.959	-0.024	-0.038	-0.012
-0.083	-0.041						

ROW(38)

-0.002	-0.004	-0.001	-0.002	-0.001	-0.001	-0.001	-0.003
-0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	-0.001	0.0	-0.001	0.0	-0.001	-0.001
-0.001	-0.001	-0.002	0.0	-0.002	0.999	-0.001	0.0
-0.003	-0.002						

```
                                  ROW(39)
-0.003 -0.008 -0.003 -0.004 -0.003 -0.002 -0.003 -0.007
-0.002 -0.001 -0.001 -0.001  0.0    0.0    0.0    0.0
 0.0    0.0   -0.001 -0.001 -0.001  0.0    0.0    0.0
 0.0    0.0   -0.001  0.0   -0.001  0.0   -0.003 -0.002
-0.002 -0.002 -0.003 -0.001 -0.003 -0.002  0.997 -0.001
-0.006 -0.003
                                  ROW(40)
-0.003 -0.009 -0.003 -0.004 -0.003 -0.002 -0.003 -0.007
-0.002 -0.001 -0.001 -0.001  0.0    0.0    0.0    0.0
 0.0    0.0   -0.001 -0.001 -0.001  0.0    0.0    0.0
 0.0    0.0   -0.001  0.0   -0.001  0.0   -0.003 -0.002
-0.003 -0.002 -0.003 -0.001 -0.003 -0.002 -0.003  0.999
-0.007 -0.003
                                  ROW(41)
-0.008 -0.021 -0.007 -0.010 -0.007 -0.005 -0.007 -0.017
-0.005 -0.002 -0.002 -0.002  0.0   -0.001  0.0   -0.001
 0.0    0.0   -0.002 -0.001 -0.002  0.0    0.0    0.0
-0.007  0.0   -0.003  0.0   -0.004 -0.001 -0.007 -0.005
-0.006 -0.004 -0.008 -0.002 -0.008 -0.005 -0.007 -0.002
 0.984 -0.008
                                  ROW(42)
 0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
 0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
 0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
 0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
 0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
 0.0    1.000
```

TABLE II. FISCAL YEAR 1971 OUTPUT REQUIREMENTS (MANHOURS) AND FISCAL YEAR 1970 NORMALIZED COST CENTER EXPENDITURES (DOLLARS) FOR THE NAVAL POSTGRADUATE SCHOOL

VECTOR 1

2857.300	6097.199	5549.098	3811.100	4488.000	10787.797
0.0	9189.000	9531.500	10004.000	45634.000	66907.500
10584.500	71291.000	25873.000	22696.000	14196.000	37419.500
61955.000	20338.000	37561.000	0.0	49949.078	23760.000
0.0	0.0	0.0	0.0	0.0	134166.063
24071.078	94604.500	29964.000	3429.000	27058.398	41913.277
94257.375	3861.210	7118.719	7690.758	18350.680	14564.316

VECTOR 2

7.476	7.100	6.601	7.097	7.089	6.063
5.278	6.458	9.289	7.194	8.969	8.716
9.376	6.934	8.366	6.648	8.813	8.429
8.170	7.938	4.343	11.166	7.781	8.983
4.260	6.549	4.726	6.031	5.094	10.189
5.262	4.260	3.725	9.121	4.852	3.369
2.614	8.274	9.451	4.407	3.482	10.014

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13. ABSTRACT

A general cost model for an educational institution is formulated. This model is developed by applying classical Leontieff input-output procedures in a situation where multiple outputs and resource inputs exist. The expansion of the classical Leontieff model to the tableau format of the "Electric" Five Year Defense Plan is presented. The general assumptions relating to input-output models are presented and analyzed in the general educational setting. To provide an example of possible uses of the model, it is applied to the Naval Postgraduate School and limited empirical results are presented. Shortcomings revealed during this first empirical use of the model are discussed.

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Cost Models						
"Electric" Five-Year Defense Plan						
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An exploratory approach to the formulation of input-output models and the application of input-output procedures in educational cost modeling.